3D DYADIC GREEN'S FUNCTION FOR RADIALLY INHOMOGENEOUS CIRCULAR FERRITE CIRCULATOR

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ABSTRACT

Here we develop a three dimensional (3D) dyadic recursive Green's function with elements G_{ij}^{sv} suitable for determining the electric field components (E_z, E_r, E_ϕ) and magnetic field components (H_z, H_r, H_ϕ) anywhere within a circular, planar (microstrip or stripline) circulator. All of the components are present, although the E_z, H_r, and H_ϕ components may still be dominant as long as the thickness h of the substrate is small. The recursive nature of G_{ij}^{sv} is a reflection of the inhomogeneous region being broken up into one inner disk containing a singularity and N annuli. $G_{ij}^{sv}(r, \phi, z)$ is found for any arbitrary point (r, ϕ, z) within the disk region and within any i th annulus. Specification of G_{ij}^{sv} , $i = E$ or H, j $= H$, $s = z$, $v = \phi$ or z, at the circulator diameter $r = R$ leads to the determination of the circulator s-parameters . The ports have been separated into discretized ports with elements (subports) and continuous ports. It is shown how $G_{ij}^{zv}(r, \phi, z)$ enables s-parameters to be found for three and six port ferrite circulators. Because of the z-variation present in the finite thickness model, TEM, TM, and TE modal decompositions are not allowed for the 3D analysis, and instead it is found that new coupled governing equations describe the field behavior in the circulator. The theory is readily adaptable to constructing a computer code for numerical evaluation of finite thickness devices. Also, symmetric port disposition and metallic losses are covered.

INTRODUCTION

As discussed in a previous paper [1], the ferrite research and development community, which has focused on producing ferrite based circulators, has been in need of simple but accurate ways of calculating performance when the device is subject to radial variation of the bias field H_{app}, ferrite material magnetization $4\pi M_s$, and demagnetization factor N_d. The two dimensional recursive Green's function employed in [1] allowed the inhomogeneous boundary valued problem, subject to inhomogeneities in parameters, to be solved in an orderly and systematic fashion. It utilized an integral-discretization mapping operator and finally resulted in scattering parameters being expressed for a three port circulator with unsymmetrically disposed ports. The theory requires the circulator region to be broken up into two different zones. The inner zone is made up of a disk containing the origin point and the outer zone is segmented or divided up into annuli, each one of unequal radial extent, layered as in an onion. Numerical calculations, based upon a FORTRAN computer code developed from the theory, show that a few seconds are required per frequency point to obtain results including s-parameters.

In contrast, two and three dimensional finite element (FE) and finite difference (FD) analyses are hundreds to thousands of times slower. Because of the success of the 2D approach, we develop here a completely new dyadic Green's function theory to describe the fields within a circulator of finite thickness. Integral-discretization operators will be employed, and spectral summation over the doubly infinite domain of azimuthal integers n will be maintained. However, because of the 3D nature of the construction, neglect of some of the field components won't be necessary any more. Although most circulators are built to be thin in terms of electrical wavelengths compared to their planar extent, assumptions requiring the thickness h to approach zero to apply a 2D model will no longer be required. Thus the actual effects of a finite thickness substrate on the

circulator behavior will now be possible. The one characteristic radial propagation constant found in the 2D model now will break up into two radial propagation constants, both affected by the allowed normal z-directed propagation constant k_z. The problem is no longer reducible to or described by a single governing equation, but rather by two coupled governing equations which always stitch the field components together. Thus, TEM, TM, and TE modes are not allowed in relation to any coordinates and no coordinate transformations will ever allow such modes to be found. Because of the impossibility of finding such simple modes, a much more involved approach to solving the 3D problem must be enlisted. Eventually, we show how these new dyadic Green's functions may be utilized to determine circulator s-parameters.

3D THEORY

The theory, briefly, starts with coupled radial Helmholtz equations stated in streamlined form as (see Figure 1 for perspective sketch of the circulator, with thickness in the z - direction exaggerated)

$$\nabla_t^2 E_z + a E_z + b H_z = 0 \tag{1a}$$

$$\nabla_t^2 H_z + c H_z + d E_z = 0 \tag{1b}$$

where

$$a = k_e^2 - k^2 \tag{2a}$$

$$b = - i\omega\mu_0 k_z \frac{\kappa}{\mu} \tag{2b}$$

$$c = \frac{\mu_0}{\mu}\left(k^2 - k_z^2\right) \tag{2c}$$

$$d = i\omega\epsilon k_z \frac{\kappa}{\mu} \tag{2d}$$

These formulas are consistent with Van Trier's earlier work for simultaneous permittivity and permeability tensors, although of specific anisotropy characteristic similar to that here resulting form a z - directed biasing magnetic field [2].

By using a diagonalization procedure appropriate to the problem, a final form of the transformed governing equation is obtained,

$$\nabla_t^2 \mathbf{F}' + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \mathbf{F}' = 0 \tag{3}$$

This diagonalization procedure is in agreement with Kales' work on ferrite wwaveguides [3], [4]. Thorough discussions of how this earlier work relates to the present circulator theory is treated elsewhere [5]. The radial field behavior is now described by the two components of the transformed field

$$\nabla_t^2 F_1' + \lambda_1 F_1' = 0 \tag{4a}$$

$$\nabla_t^2 F_2' + \lambda_2 F_2' = 0 \tag{4b}$$

These equations can be solved independently, just as one would solve a single Helmholtz equation in cylindrical coordinates. Thus the approach utilized for the two dimensional circulator problem [6] can be used to good advantage here. Of course, the actual field component solutions in the z-direction must be found by a proper linear superposition given by the reverse transformations for E_z and H_z. From these two properly determined field component solutions, all of the other transverse field components in the plane of the circulator can be also acquired by the construction technique.

$$E_z = F_1' + F_2' \qquad (5a)$$

$$H_z = \frac{\lambda_1 - a}{b} F_1' + \frac{\lambda_2 - a}{b} F_2' \qquad (5b)$$

The transverse field component formulas, in terms of the transformed fields and the radial eigenvalues, are

$$E_r = -\frac{r\lambda_2}{b}\frac{\partial F_1'}{\partial r} - \frac{i\omega\mu_0 + s\lambda_2}{br}\frac{\partial F_1'}{\partial \phi} - \frac{r\lambda_1}{b}\frac{\partial F_2'}{\partial r} - \frac{i\omega\mu_0 + s\lambda_1}{br}\frac{\partial F_2'}{\partial \phi} \qquad (6a)$$

$$E_\phi = -\frac{r\lambda_2}{br}\frac{\partial F_1'}{\partial \phi} + \frac{i\omega\mu_0 + s\lambda_2}{b}\frac{\partial F_1'}{\partial r} - \frac{r\lambda_1}{br}\frac{\partial F_2'}{\partial \phi} + \frac{i\omega\mu_0 + s\lambda_1}{b}\frac{\partial F_2'}{\partial r} \qquad (6b)$$

$$H_r = \frac{ik_z(\mu_0/\mu) - p\lambda_2}{b}\frac{\partial F_1'}{\partial r} - \frac{q\lambda_2}{br}\frac{\partial F_1'}{\partial \phi} + \frac{ik_z(\mu_0/\mu) - p\lambda_1}{b}\frac{\partial F_2'}{\partial r} - \frac{q\lambda_1}{br}\frac{\partial F_2'}{\partial \phi} \qquad (7a)$$

$$H_\phi = \frac{ik_z(\mu_0/\mu) - p\lambda_2}{br}\frac{\partial F_1'}{\partial \phi} + \frac{q\lambda_2}{b}\frac{\partial F_1'}{\partial r} + \frac{ik_z(\mu_0/\mu) - p\lambda_1}{br}\frac{\partial F_2'}{\partial \phi} + \frac{q\lambda_1}{b}\frac{\partial F_2'}{\partial r} \qquad (7b)$$

The field components can now be constructed in terms of the dyadic Green's functions, when the circulator is considered driven by the external ports. For example, the z-component of the electric field in the disk region is

$$E_{z0}(r, \phi, z) = \sum_{s=1}^{N_z}\sum_{q=1}^{N_{Tp}^d}\sum_{k=1}^{N_p^q} G_{EH0}^{z\phi}\left(r, \phi, z; R, \phi_k^q, z_s\right)H_{\phi c}\left(R, \phi_k^q, z_s\right)\Delta\phi_k^q$$

$$+ \sum_{s=1}^{N_z}\sum_{v=1}^{N_{Tp}^d} \overline{G}_{EH0}^{z\phi}\left(r, \phi, z; R, \phi_v, z_s\right)H_{\phi c}\left(R, \phi_v, z_s\right)\Delta\phi_k$$

$$+ \sum_{s=1}^{N_z}\sum_{q=1}^{N_{Tp}^d}\sum_{k=1}^{N_p^q} G_{EH0}^{zz}\left(r, \phi, z; R, \phi_k^q, z_s\right)H_{zc}\left(R, \phi_k^q, z_s\right)\Delta\phi_k^q$$

$$+ \sum_{s=1}^{N_z}\sum_{v=1}^{N_{Tp}^d} \overline{G}_{EH0}^{zz}\left(r, \phi, z; R, \phi_v, z_s\right)H_{zc}\left(R, \phi_v, z_s\right)\Delta\phi_k \qquad (8)$$

Here, the first dyadic Green's function in the expansion is given by

$$G_{EH0}^{z\phi} = \frac{1}{2\pi}\sum_{j=0}^{\infty}\sum_{n=-\infty}^{\infty} K_{zj+}\cos(k_{z0j+}z)\frac{1}{D_{ABj}}$$

$$\times\left[B_{nj}^2 C_{neaj0}^{z1}(r) - B_{nj}^1 C_{neaj0}^{z2}(r)\right]e^{-in\phi_k^q}e^{in\phi} \qquad (9)$$

This dyadic Green's function properly superpositions the k_z modal dependence, and allows the correct coupling of the device to external port fields.

The theory can treat any number of ports, discretizing them if desired as done for the general 2D theory. Anyway, for the commonly studied 3 - port case, at the circulator - port interfaces,

$$E_a = \overset{\approx\phi}{G}_{eh}(\phi_a,\phi_a)H_{\phi a} + \overset{\approx z}{G}_{eh}(\phi_a,\phi_a)H_{za} + \overset{\approx\phi}{G}_{eh}(\phi_a,\phi_b)H_{\phi b} + \overset{\approx z}{G}_{eh}(\phi_a,\phi_b)H_{zb}$$
$$+ \overset{\approx\phi}{G}_{eh}(\phi_a,\phi_c)H_{\phi c} + \overset{\approx z}{G}_{eh}(\phi_a,\phi_c)H_{zc} \qquad (10a)$$

$$E_b = \overset{\approx\phi}{G}_{eh}(\phi_b,\phi_a)H_{\phi a} + \overset{\approx z}{G}_{eh}(\phi_b,\phi_a)H_{za} + \overset{\approx\phi}{G}_{eh}(\phi_b,\phi_b)H_{\phi b} + \overset{\approx z}{G}_{eh}(\phi_b,\phi_b)H_{zb}$$
$$+ \overset{\approx\phi}{G}_{eh}(\phi_b,\phi_c)H_{\phi c} + \overset{\approx z}{G}_{eh}(\phi_b,\phi_c)H_{zc} \qquad (10b)$$

$$E_c = \overset{\approx\phi}{G}_{eh}(\phi_c,\phi_a)H_{\phi a} + \overset{\approx z}{G}_{eh}(\phi_c,\phi_a)H_{za} + \overset{\approx\phi}{G}_{eh}(\phi_c,\phi_b)H_{\phi b} + \overset{\approx z}{G}_{eh}(\phi_c,\phi_b)H_{zb}$$
$$+ \overset{\approx\phi}{G}_{eh}(\phi_c,\phi_c)H_{\phi c} + \overset{\approx z}{G}_{eh}(\phi_c,\phi_c)H_{zc} \qquad (10c)$$

$$H_{za} = \overset{\approx\phi}{G}_{hh}(\phi_a,\phi_a)H_{\phi a} + \overset{\approx z}{G}_{hh}(\phi_a,\phi_a)H_{za} + \overset{\approx\phi}{G}_{hh}(\phi_a,\phi_b)H_{\phi b} + \overset{\approx z}{G}_{hh}(\phi_a,\phi_b)H_{zb}$$
$$+ \overset{\approx\phi}{G}_{hh}(\phi_a,\phi_c)H_{\phi c} + \overset{\approx z}{G}_{hh}(\phi_a,\phi_c)H_{zc} \qquad (11a)$$

$$H_{zb} = \overset{\approx\phi}{G}_{hh}(\phi_b,\phi_a)H_{\phi a} + \overset{\approx z}{G}_{hh}(\phi_b,\phi_a)H_{za} + \overset{\approx\phi}{G}_{hh}(\phi_b,\phi_b)H_{\phi b} + \overset{\approx z}{G}_{hh}(\phi_b,\phi_b)H_{zb}$$
$$+ \overset{\approx\phi}{G}_{hh}(\phi_b,\phi_c)H_{\phi c} + \overset{\approx z}{G}_{hh}(\phi_b,\phi_c)H_{zc} \qquad (11b)$$

$$H_{zc} = \overset{\approx\phi}{G}_{hh}(\phi_c,\phi_a)H_{\phi a} + \overset{\approx z}{G}_{hh}(\phi_c,\phi_a)H_{za} + \overset{\approx\phi}{G}_{hh}(\phi_c,\phi_b)H_{\phi b} + \overset{\approx z}{G}_{hh}(\phi_c,\phi_b)H_{zb}$$
$$+ \overset{\approx\phi}{G}_{hh}(\phi_c,\phi_c)H_{\phi c} + \overset{\approx z}{G}_{hh}(\phi_c,\phi_c)H_{zc} \qquad (11c)$$

S - PARAMETERS

These relationships can be used to find the s-parameters of the 3D modelled device. The solutions for the H-field components are

$$H_{\phi a} = \frac{1}{D_{3p}}\begin{vmatrix} 2 & T_{ab}^e & T_{ac}^e \\ 0 & \left(T_{bb}^e + \zeta_b^e\right) & T_{bc}^e \\ 0 & T_{cb}^e & \left(T_{cc}^e + \zeta_c\right) \end{vmatrix} \qquad (12a)$$

$$H_{\phi b} = \frac{1}{D_{3p}}\begin{vmatrix} \left(T_{aa}^e + \zeta_a\right) & 2 & T_{ac}^e \\ T_{ba}^e & 0 & T_{bc}^e \\ T_{ca}^e & 0 & \left(T_{cc}^e + \zeta_c\right) \end{vmatrix} \qquad (12b)$$

$$H_{\phi c} = \frac{1}{D_{3p}}\begin{vmatrix} \left(T_{aa}^e + \zeta_a\right) & T_{ab}^e & 2 \\ T_{ba}^e & \left(T_{bb}^e + \zeta_b\right) & 0 \\ T_{ca}^e & G_{cb}^e & 0 \end{vmatrix} \qquad (12c)$$

where the H - field system determinant is

$$D_{3p} = \begin{vmatrix} \left(T_{aa}^e + \zeta_a\right) & T_{ab}^e & T_{ac}^e \\ T_{ba}^e & \left(T_{bb}^e + \zeta_b\right) & T_{bc}^e \\ T_{ca}^e & T_{cb}^e & \left(T_{cc}^e + \zeta_c\right) \end{vmatrix} \qquad (13)$$

A typical group of T elements may look like

$$T_{aa}^h = -G_{hh}^\phi(aa)M_{aa} + G_{hh}^\phi(ba)M_{ab} - G_{hh}^\phi(ca)M_{ac} \qquad (14a)$$

$$T_{ab}^h = -G_{hh}^\phi(ab)M_{aa} + G_{hh}^\phi(bb)M_{ab} - G_{hh}^\phi(cb)M_{ac} \qquad (14b)$$

$$T_{ac}^h = -G_{hh}^\phi(ac)M_{aa} + G_{hh}^\phi(bc)M_{ab} - G_{hh}^\phi(cc)M_{ac} \qquad (14c)$$

where

$$M_{aa} = \begin{vmatrix} G_{hh}^Z(bb) - 1 & G_{hh}^Z(bc) \\ G_{hh}^Z(cb) & G_{hh}^Z(cc) - 1 \end{vmatrix} \qquad (15a)$$

$$M_{ab} = \begin{vmatrix} G_{hh}^Z(ab) & G_{hh}^Z(ac) \\ G_{hh}^Z(cb) & G_{hh}^Z(cc) - 1 \end{vmatrix} \qquad (15b)$$

$$M_{ac} = \begin{vmatrix} G_{hh}^Z(ab) & G_{hh}^Z(ac) \\ G_{hh}^Z(bb) - 1 & G_{hh}^Z(bc) \end{vmatrix} \qquad (15c)$$

Once the perimeter fields have been determined, it is a simple matter to apply the prescription employed in the 2D model, here to the 3D s - parameter calculation. The three s - parameters are then

$$s_{11} = 1 - \zeta_a H_{\phi a} \tag{16a}$$

$$s_{21} = - \zeta_b H_{\phi b} \tag{16b}$$

$$s_{31} = - \zeta_c H_{\phi c} \tag{16c}$$

The counter - clockwise labelling scheme of the ports is shown in Fig. 2.

THREE - PORT SYMMETRIC 2D CIRCULATORS

There are a tremendous amount of simplifications which result by constraining the circulator to a symmetric disposition of the port locations. We use the 2D theory as illustrative, leaving the details of the 3D exposition for another place.

$$E_a = G_{aa} H_a + G_{ab} H_b + G_{ac} H_c \tag{17a}$$

$$E_b = G_{ba} H_a + G_{bb} H_b + G_{bc} H_c \tag{17b}$$

$$E_c = G_{ca} H_a + G_{cb} H_b + G_{cc} H_c \tag{17c}$$

In the general case for the 2D model, (17) holds for a three - port device. Examination of the EH dyadic Green's function element for n = N, the last ring, allows full advantage to be taken of the inherent three-fold symmetry.

$$G_{EHN}^{z\phi}(R, \phi; R, \phi_k^q) = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{nN}^{ze} e^{-in\phi_k^q} e^{in\phi} \tag{18}$$

Let us make this expression more transparent by defining the source azimuthal location to be $\phi_j = \phi_k^q$ and the field location to be $\phi = \phi_i$. Furthermore, abbreviate

$$G_{EHN}^{z\phi}(R, \phi; R, \phi_k^q) \Big|_{\substack{\phi = \phi_i \\ \phi_k^q = \phi_j}} = G(\phi_i, \phi_j) \tag{19}$$

On the right-hand-side of (18), collect the azimuthal exponents into one factor, while noting that the radial variation, here with r = R, is stored in the pre-factor $\overline{\gamma}_{nN}^{ze} = \overline{\gamma}_{nN}^{ze}(R)$. Then (18) becomes

$$G(\phi_i, \phi_j) = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{nN}^{ze} e^{in(\phi_i - \phi_j)} \tag{20}$$

Right away, we notice from (20) that the Green's function only depends upon the difference between the source and field locations. That is,

$$G(\phi_i, \phi_j) = G(\phi_i - \phi_j) \tag{21}$$

This does not mean, however, that $G(\phi_i, \phi_j) = G(\phi_j, \phi_i)$. In fact, because the material is a nonreciprocating medium, we know this can't hold. Certainly, circulating action would cease if this type of Green's function dyadic element symmetry existed. Another clue that this type of symmetry doesn't exist is seen by re-examining (20) again. Let us find the $G(\phi_j, \phi_i)$ dyadic element from (20) by switching azimuthal angles.

$$G(\phi_j, \phi_i) = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{nN}^{ze} e^{in(\phi_j - \phi_i)} \tag{22}$$

Now define a new summation index as n' = - n. Substituting this into (6) yields

$$G(\phi_j, \phi_i) = \frac{1}{2\pi} \sum_{n' = \infty}^{-\infty} \overline{\gamma}_{-n'N}^{ze} e^{-in'(\phi_i - \phi_j)} \tag{23}$$

But because notation is arbitrary, change the n' into n for the index, and recognize that the order of summation doesn't matter, especially as the same integers are used as in the original $G(\phi_i, \phi_j)$ Green's function.

$$G(\phi_j, \phi_i) = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{-nN}^{ze} e^{in(\phi_i - \phi_j)} \tag{24}$$

Comparing this to the expression for $G(\phi_i, \phi_j)$ in (20) enables us to see that except for the pre-factor, the dyadic Green's function formulas are

identical. It is this pre-factor, nevertheless, which is all important here and maintains the device non-reciprocity! $\overline{\gamma}_{-nN}^{ze}$ is not equal to $\overline{\gamma}_{nN}^{ze}$, or

$$\overline{\gamma}_{-nN}^{ze} \neq \overline{\gamma}_{nN}^{ze} \tag{25}$$

Because of the all important relationship in (25),

$$G(\phi_j, \phi_i) \neq G(\phi_i, \phi_j) \tag{26}$$

is true and we are assured of our non-reciprocity.

However, property (21) and the symmetric angular distribution of the port locations will reduce the number of actual Green's function elements required to be calculated. Note that the notation

$$G_{ij} = G(\phi_i, \phi_j) \tag{27}$$

is used in (17). Thus, by evaluating the Green's dyadic for $\phi_i = \phi_j$, the self - terms, it is found that

$$G(\phi_i, \phi_i) = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{nN}^{ze} \tag{28}$$

All self-term Green's function dyadic elements are equal, and we denote this fact by assigning

$$G_s = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{nN}^{ze} \tag{29}$$

Equation (28) obviously also means that

$$G(\phi_i, \phi_i) = G(\phi_j, \phi_j) \tag{30}$$

For the off - diagonal Green's function dyadic elements, it is found that the number of unique terms is less than the total number of off - diagonal elements. Denote the total number of radially located ports as N_{Trp}. Then the total number of diagonal elements is N_T, diag $= N_{Trp}$, but the number of unique diagonal elements is one by the arguments in the previous paragraph. The total number of off - diagonal elements is N_T, off - diag $= (N_{Trp})^2 - N_{Trp}$, but the number of unique diagonal elements is considerably less, N_{unique}, off - diag $= N_{Trp}^{even}$ for an even number of ports and N_{unique}, off - diag $= N_{Trp}^{odd} - 1$ for an odd number of ports. A device with three ports would have $N_{u, o - d} = 2 < N_T$, off - diag $= 6$. The savings for larger N_{Trp} rapidly goes up because of the quadratic term.

The determinant of the system to solve for the azimuthal H_ϕ - fields also displays all the individual Green's function dyadic elements. (How the G entries of the 2D theory go over into T entries of the 3D theory is discussed in [5].)

$$D_p = \begin{vmatrix} (G_{aa} + \zeta_a) & G_{ab} & G_{ac} \\ G_{ba} & (G_{bb} + \zeta_b) & G_{bc} \\ G_{ca} & G_{cb} & (G_{cc} + \zeta_c) \end{vmatrix} \tag{31}$$

Let us define

$$\Delta\phi_{ij} = \phi_i - \phi_j \tag{32}$$

Recognizing that a right-handed system with (r, ϕ) or (x, y) in the plane of the paper requires counter - clockwise labeled ports (i.e., i = a, b, c or i = 1, 2, 3) to have progressively more positive valued azimuthal angles, if the input port angle is set to 0 radians, then

$$\phi_a = 0 \; ; \; \phi_b = 2\pi/3 \; ; \; \phi_c = 4\pi/3 \tag{33}$$

In order to see that there are only two unique dyadic Green's function elements requiring calculation, it is best to begin to evaluate the particular off - diagonal terms in (31). It will become apparent what the trend is once this examination process is started. By (20), (27), and (32),

$$G_{ij} = G_{ij}(\Delta\phi_{ij}) = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{nN}^{ze} e^{in\Delta\phi_{ij}} \tag{34}$$

Moving down the first column, excluding the diagonal, the ij = 21 or ba element is

$$G_{ba} = G_{ba}(\Delta\phi_{ba}) = \frac{1}{2\pi} \sum_{n = -\infty}^{\infty} \overline{\gamma}_{nN}^{ze} e^{in\Delta\phi_{ba}} \tag{35}$$

invoking (33),

$$\Delta\phi_{ba} = \phi_b - \phi_a = 2\pi/3 \tag{36}$$

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and inserting this into (35) yields one of the unique dyadic Green's function elements.

$$G_+ = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \overline{\gamma}_{nN}^{ze}\ e^{2in\pi/3} \tag{37}$$

with
$$G_{ba} = G_+ \tag{38}$$

Moving down to the next element in the system matrix, the 31 or ca element, we find the angular argument

$$\Delta\phi_{ca} = \phi_c - \phi_a = 4\pi/3 = 2\pi - 2\pi/3 \tag{39}$$

and put it into

$$G_{ca} = G_{ca}(\Delta\phi_{ca}) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \overline{\gamma}_{nN}^{ze}\ e^{in\Delta\phi_{ca}} \tag{40}$$

obtaining the other unique dyadic Green's function element.

$$G_- = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \overline{\gamma}_{nN}^{ze}\ e^{-2in\pi/3} \tag{41}$$

with
$$G_{ca} = G_- \tag{42}$$

The dyadic Green's function elements in the first row in (31), are found from those already determined by noting that the azimuthal angle differences have their signs reversed. Finally the cb (and bc) element is determined once it is noted that

$$\Delta\phi_{cb} = \phi_c - \phi_b = 4\pi/3 - 2\pi/3 = 2\pi/3 \tag{43}$$

By (34), this implies that the cb element is G_- . Therefore, in summary we have found that

$$G_{aa} = G_{bb} = G_{cc} = G_s \tag{44a}$$
$$G_{ca} = G_{bc} = G_{ab} = G_- \tag{44b}$$
$$G_{ba} = G_{cb} = G_{ac} = G_+ \tag{44c}$$

Placing (32) into (19) gives

$$D_p = D_{3\times 3} = \begin{vmatrix} (G_s + \zeta_a) & G_- & G_+ \\ G_+ & (G_s + \zeta_b) & G_- \\ G_- & G_+ & (G_s + \zeta_c) \end{vmatrix} \tag{45}$$

Now the three azimuthal H - fields can be simplified as follows:

$$H_a = \frac{2}{D_p}\left[(G_{bb} + \zeta_b)(G_{cc} + \zeta_c) - G_{bc}G_{cb}\right]$$
$$= \frac{2}{D_p}\left[(G_s + \zeta_b)(G_s + \zeta_c) - G_-G_+\right] \tag{46a}$$

$$H_b = -\frac{2}{D_p}\left[G_{ba}(G_{cc} + \zeta_c) - G_{bc}G_{ca}\right]$$
$$= -\frac{2}{D_p}\left[G_+(G_s + \zeta_c) - [G_+]^2\right] \tag{46b}$$

$$H_c = \frac{2}{D_p}\left[G_{ba}G_{cb} - (G_{bb} + \zeta_b)G_{ca}\right]$$
$$= \frac{2}{D_p}\left[[G_-]^2 - (G_s + \zeta_b)G_+\right] \tag{46c}$$

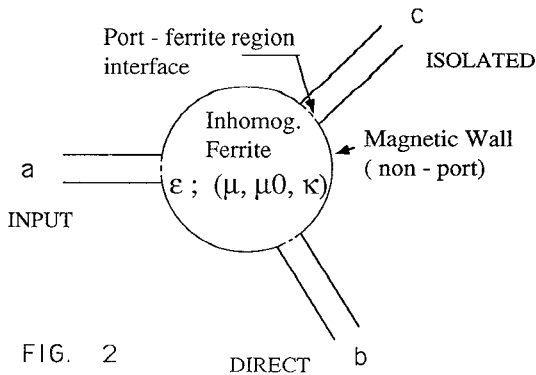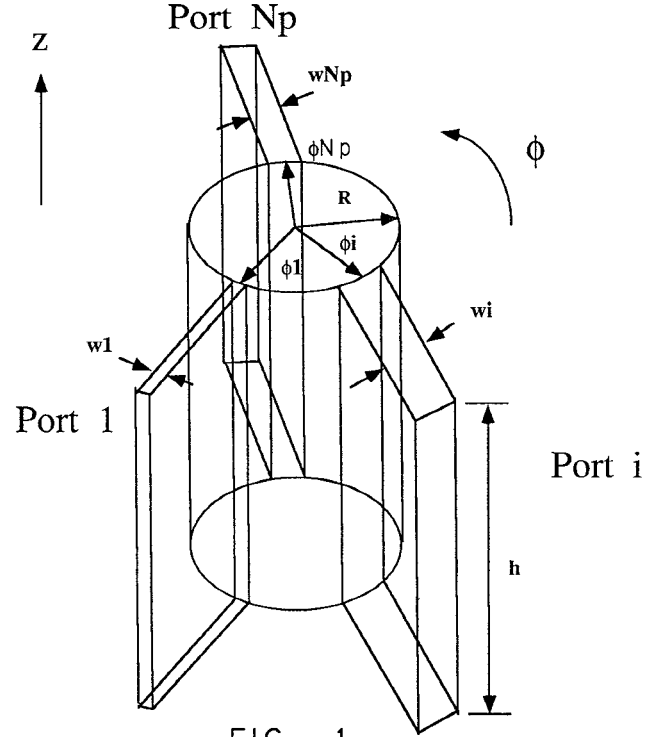The s-parameters are found in the usual manner, from (16) above.

FIG. 2

FIG. 1

METALLIC WALL LOSSES IN 3D MODEL

Rigorous incorporation of metal ground plane and microstrip losses, something missing in the 2D approach, by the use of surface impedances at the two conductor surfaces in question, can readily be accomplished. By restricting the mixed recursive dyadic Green's function /mode - matching approach in [5] to the limiting case of one horizontal layer, thereby reducing the RGF/MM to a RGF, but retaining the added constraint imposed to describe the impedance boundary conditions

$$Z_s^m J_{sm} = E_s^m \tag{47}$$

found there, where m = layer index, which here would be set to m = 1B or 1T, corresponding to the bottom or top part of the layer (here just a single region), losses may be added. Z_s^m in [5] is used as a scalar, but may be upgraded to a dyadic if desired.

CONCLUSIONS

It is expected that the derived dyadic Green's function will be very useful for the calculation of the 3D circulator fields and s-parameters because of the previously found rapid convergence properties of the Bessel function expressions previously used for the 2D circulator and similarly employed here.

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